Technical E-Paper

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Introduction to Array Antennas

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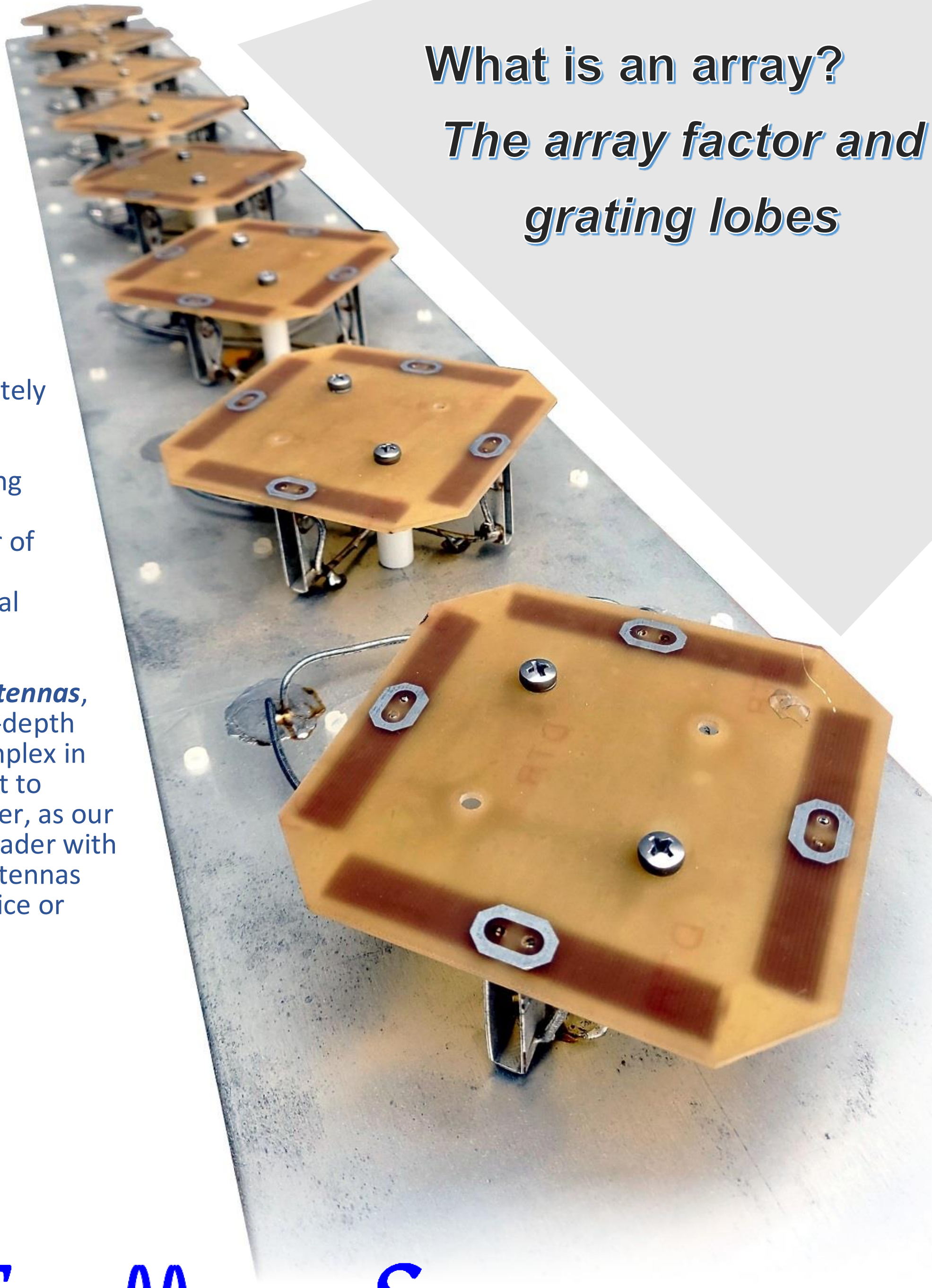
What is an array? The array factor and grating lobes

To function properly, a wireless system must use antennas with well-defined radiation characteristics: high gain and/or radiation patterns that allow the maximum possible energy to be radiated and received in the desired directions.

This is often achieved with antennas capable of appropriately shaping the radiation pattern according to the required specifications, i.e. by developing complex radiating systems composed of a certain number of simpler antennas, arranged in specific geometric and electrical configurations.

We are talking about ***array antennas***, the subject of this technical in-depth analysis. The topic is quite complex in itself, and here we will attempt to present it in a simplified manner, as our aim is always to provide the reader with useful tools to evaluate the antennas they use for their system, service or application.



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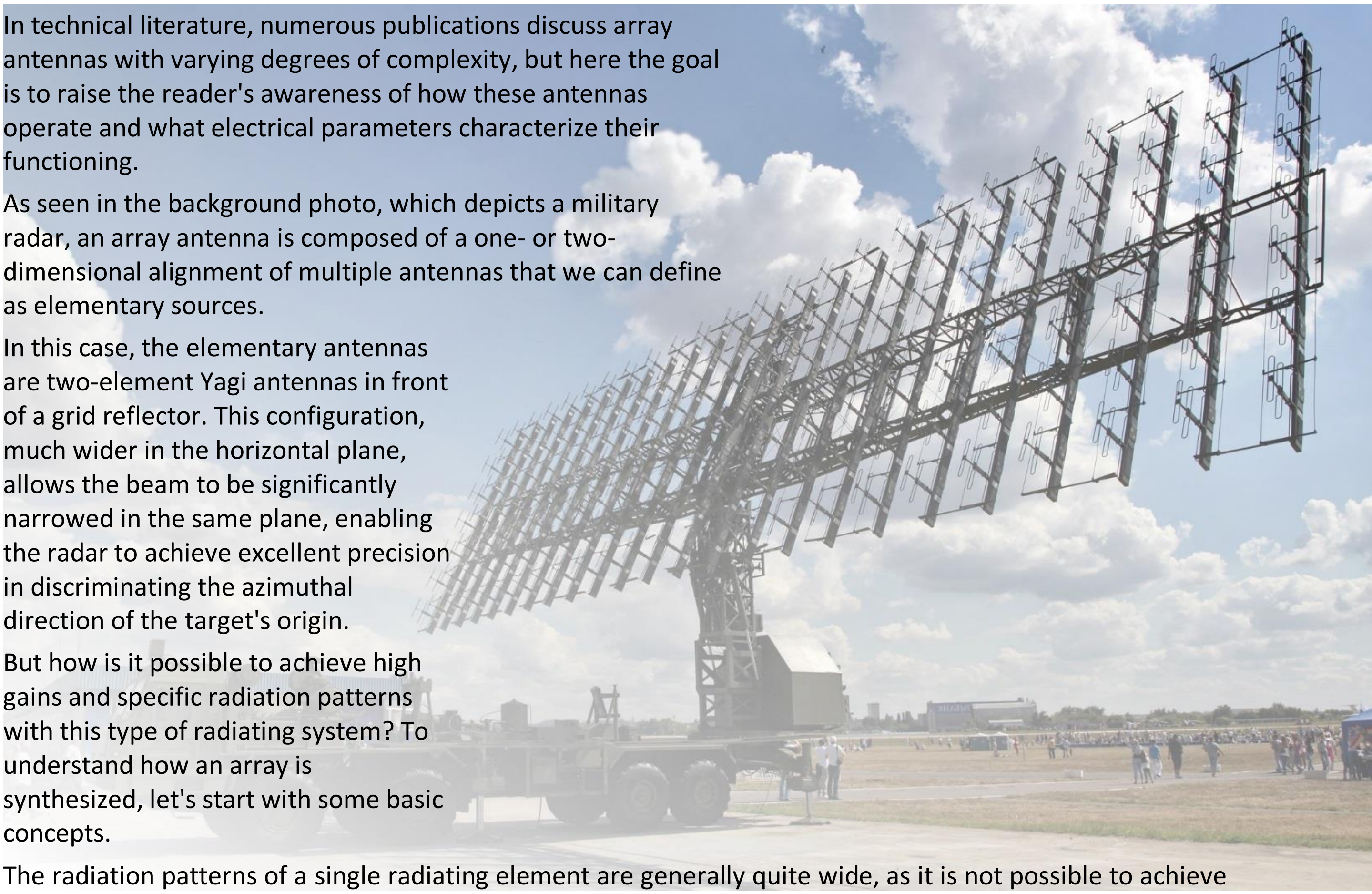
1. Introduzione.



In technical literature, numerous publications discuss array antennas with varying degrees of complexity, but here the goal is to raise the reader's awareness of how these antennas operate and what electrical parameters characterize their functioning.

As seen in the background photo, which depicts a military radar, an array antenna is composed of a one- or two-dimensional alignment of multiple antennas that we can define as elementary sources.

In this case, the elementary antennas are two-element Yagi antennas in front of a grid reflector. This configuration, much wider in the horizontal plane, allows the beam to be significantly narrowed in the same plane, enabling the radar to achieve excellent precision in discriminating the azimuthal direction of the target's origin.

But how is it possible to achieve high gains and specific radiation patterns with this type of radiating system? To understand how an array is synthesized, let's start with some basic concepts.

The radiation patterns of a single radiating element are generally quite wide, as it is not possible to achieve high gains, i.e., greater than 8–10 dBi, with a source of limited size (less than λ). If it becomes necessary to increase the directivity of the antenna, its size (in terms of λ) can be increased by creating a radiating system composed of multiple elementary antennas, arranged in an appropriate geometric and electrical configuration: this results in an **array antenna**.

In far-field conditions, i.e. at a very far distance from the array, the total radiated field is the vector sum of the individual fields produced by the sources that constitute it. Therefore, to control the radiation patterns of such a radiating system, it is possible to act on the following degrees of freedom:

- the geometric configuration of the array (linear, planar, circular, one-, two- or three-dimensional, etc.);
- the relative position of the elements, particularly their spacing ***s/λ*** in the case of uniform arrays;
- the radiation pattern of the individual radiating elements;
- the power distribution to the individual radiating elements;
- the phase shift between the individual radiating elements.

If we consider linear and planar arrays, which are respectively composed of ***N*** radiating elements placed along a line segment or ***N×M*** elements arranged in a rectangular plane region, they are classified as ***broadside*** or ***endfire*** depending on whether the direction of maximum radiation is perpendicular or parallel to the alignment of the sources.

An example of an ***endfire array*** is well-known to everyone, as the Yagi antenna, or more precisely, the Yagi-Uda array, is widely used in domestic television reception.

We will cover this antenna in a future technical analysis. Here, we will focus on ***broadside arrays***, where the direction of maximum radiation is perpendicular to the line or plane on which the individual antennas are arranged.

2. Array Antennas: directivity function and array factor.

Before examining the radiation of an array antenna, let's start by considering an ***isotropic antenna***, which radiates a power P_{irr} evenly in all directions.

At a distance *r* (in the far field), an isotropic radiator produces a power density:

$$S_{iso} = \frac{P_{irr}}{4\pi r^2} \qquad [\mathrm{W/m^2}] \qquad [1]$$

Conversely, a real antenna that radiates the same total power P_{irr} will favor certain directions over others, and equation [1] should then be understood as the average power density over all directions. The actual value will therefore be given by:

$$S = \frac{P_{irr}}{4\pi r^2} D\, f(\theta, \phi) \qquad [\mathrm{W/m^2}] \qquad [2]$$

In other words, it depends on a directional function, expressed as the product of a function *f* normalized to a maximum unit value (known as the **directivity function**) and a coefficient *D* called **directivity**. The latter is then expressed by the ratio:

$$D = \frac{S_{MAX}}{S_{iso}} \qquad [3]$$

where, in the direction of maximum radiation, $f(\theta,\phi)=1$.

If we consider an array antenna composed of a given number of identical radiating elements, all oriented in the same direction, and $f_{el}(\theta,\phi)$ is the directivity function of one of these sources, equation [2] becomes:

$$S_{array} = \frac{P_{irr}}{4\pi r^2} D\, f_{el}(\theta, \emptyset)\, f_g(\theta, \phi) \qquad [\mathrm{W/m^2}] \qquad [4]$$

where the directional function $f_g(\theta,\phi)$, also with a maximum unit value, is called the **array factor**.

Therefore, the directivity function of the array is given by:

$$f_{array}(\theta, \emptyset) = f_{el}(\theta, \emptyset)\, f_g(\theta, \phi) \qquad [5]$$

In other words, the directivity function of the array is equal to the product of the directivity function of the single element and the array factor. This latter function does not depend on the type of antennas that form the array but solely on their mutual position and how they are fed, in both amplitude and phase (i.e. the excitation coefficients).

By setting $f_{el}(\theta,\phi)=1$ in [5], the array factor can be defined as the directivity function of the array when the individual antennas that constitute it are isotropic radiators.

Therefore, by studying the properties of this function, it is possible to understand many aspects, even from an intuitive point of view, about the radiative properties of an array antenna.

2. Properties of the array factor: grating lobes.

Referring to **Figure 2.1**, let's consider a one-dimensional antenna array with a uniform spacing *d* and uniform feeding of the individual sources (the red dipoles in the figure).

A plane wave with an angle *α* relative to the alignment of the sources (the equiphase surfaces are shown in blue) will reach each of the individual sources with a different phase shift, depending on the individual path difference relative to the reference point *O*, the center of the array.

For example, for the pair of sources 1 and 2, the phase term of the electric field due to the path difference is:

$$h = \frac{d}{2} \sin\alpha$$

is given by:

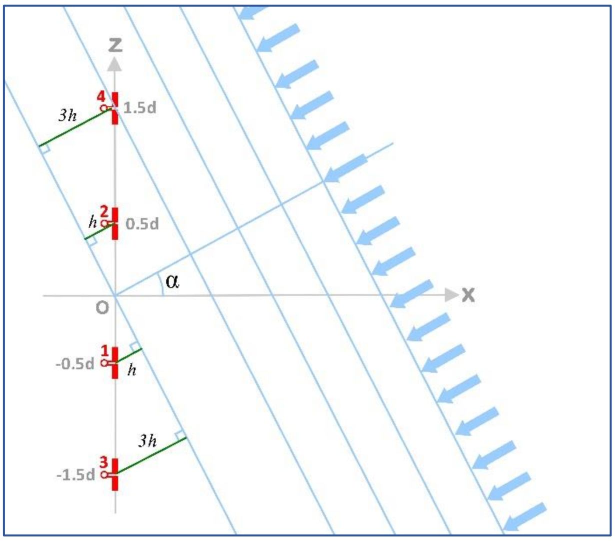


Figure 2.1
One-dimensional Array.

$$exp\left(j\frac{2\pi}{\lambda}h\right) + exp\left(-j\frac{2\pi}{\lambda}h\right) = 2\,cos\left[\pi\frac{d}{\lambda}sin\,\alpha\right] \qquad [6]$$

By repeating this reasoning for all pairs of sources symmetrical with respect to the centre O of the array, the expression for the *array factor in the (x,z) plane* of the array in **Figure 2.1** is obtained, extending the reasoning to *N/2* pairs of radiating elements (an array of order *N*):

$$f_g(\alpha) = \left\{\frac{2}{N}\sum_{n=1}^{N/2}\cos\left[\pi(2n-1)\frac{d}{\lambda}\sin\alpha\right]\right\}^2 \qquad [7]$$

Let us now avoid further mathematical details and simply examine the behavior of the array factor [7] as a function of the number *N* of array elements and their spacing *d/λ* and how equation [5] consequently changes.

To do this, let's imagine constructing an array with elementary radiating elements that have a beamwidth of about 85° in the *(x,z)* plane, as shown in **Figure 2.2**.

We then apply equation [5], considering the respective radiation patterns in the *(x,z)* plane of the one-dimensional array instead of the three-dimensional functions, as follows:

$$f_{array}(\alpha) = f_{el}(\alpha)\, f_g(\alpha)$$

By repeating this calculation for *N=2,4,8* and for different spacings *d/λ* between the elements, the array factors and radiation patterns shown in **Figures 2.3 to 2.7** are obtained.

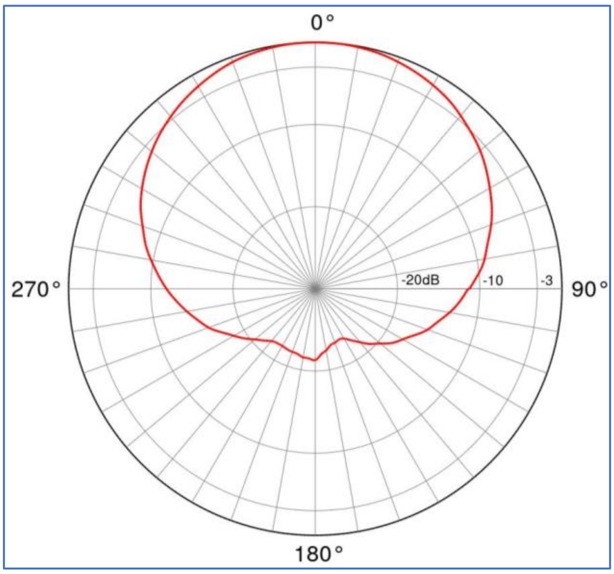
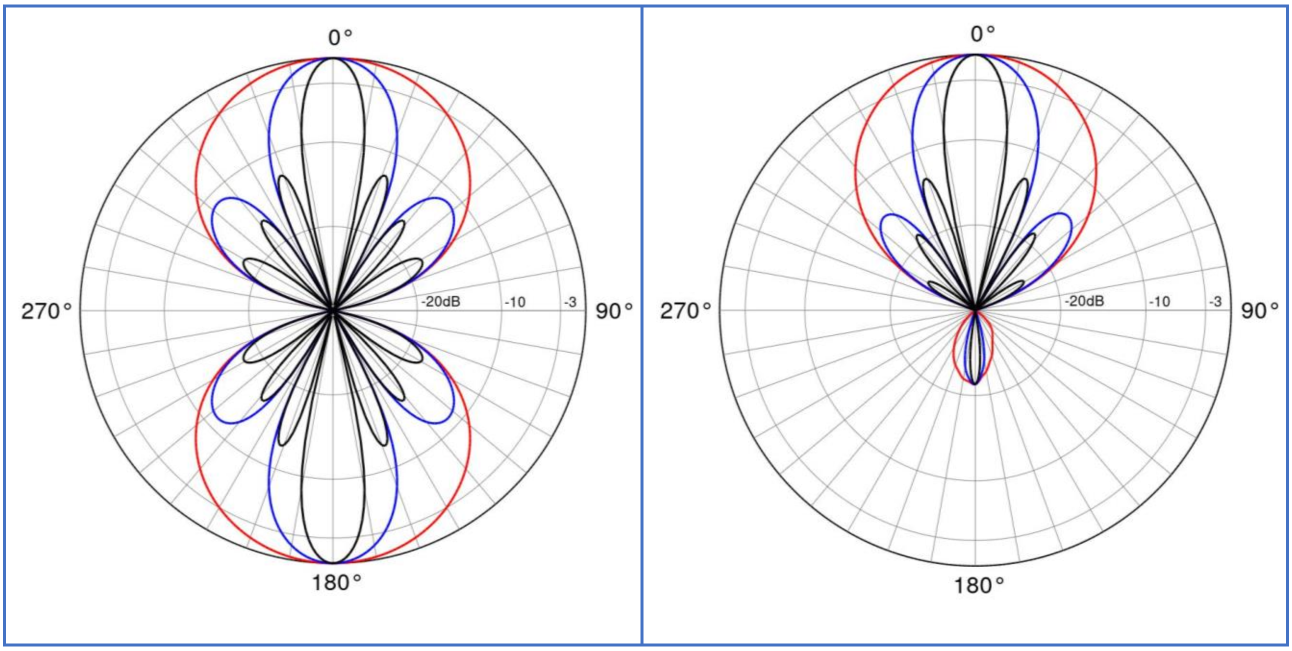


Figure 2.2
Radiation pattern of the single radiating element, i.e., the intersection of $f_{el}(\theta,\phi)$ with the *(x,z)* plane of the array.

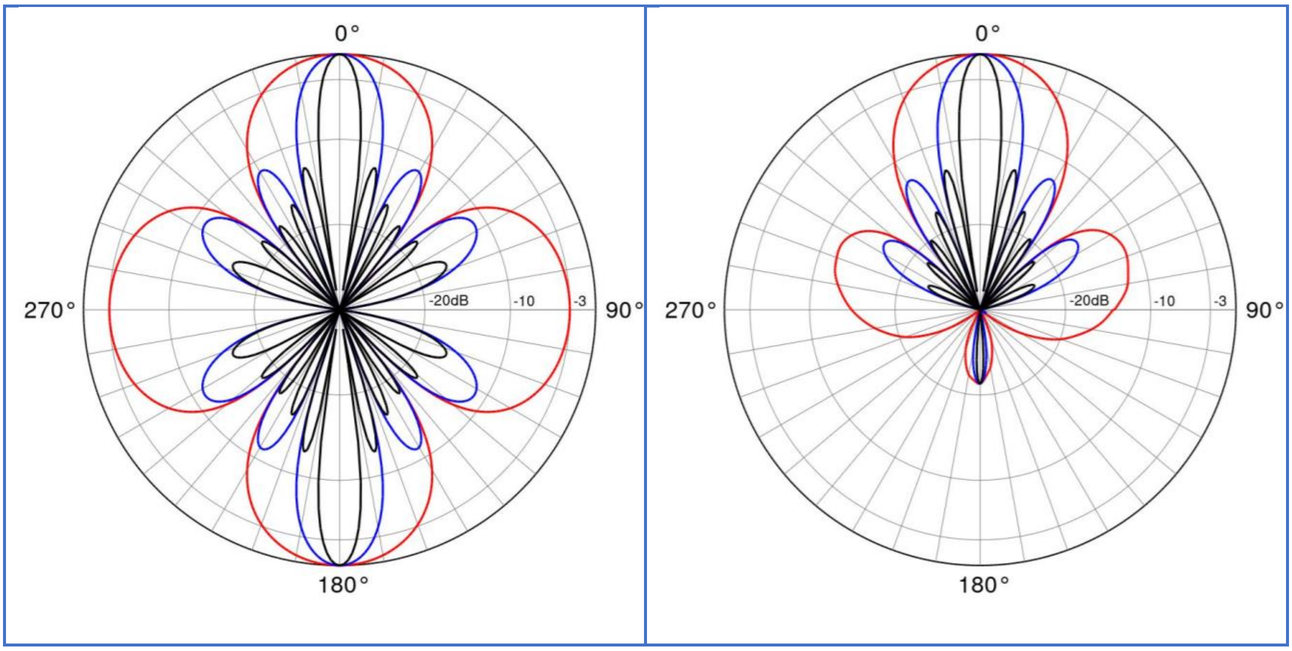


Array factor $f_g(\alpha)$ **for N=2, 4, 8**

Radiation pattern of the array $f_{array}(\alpha)$ **for N=2, 4, 8**

Figure 2.3

Array factor and radiation pattern for ***d=0.5λ***.

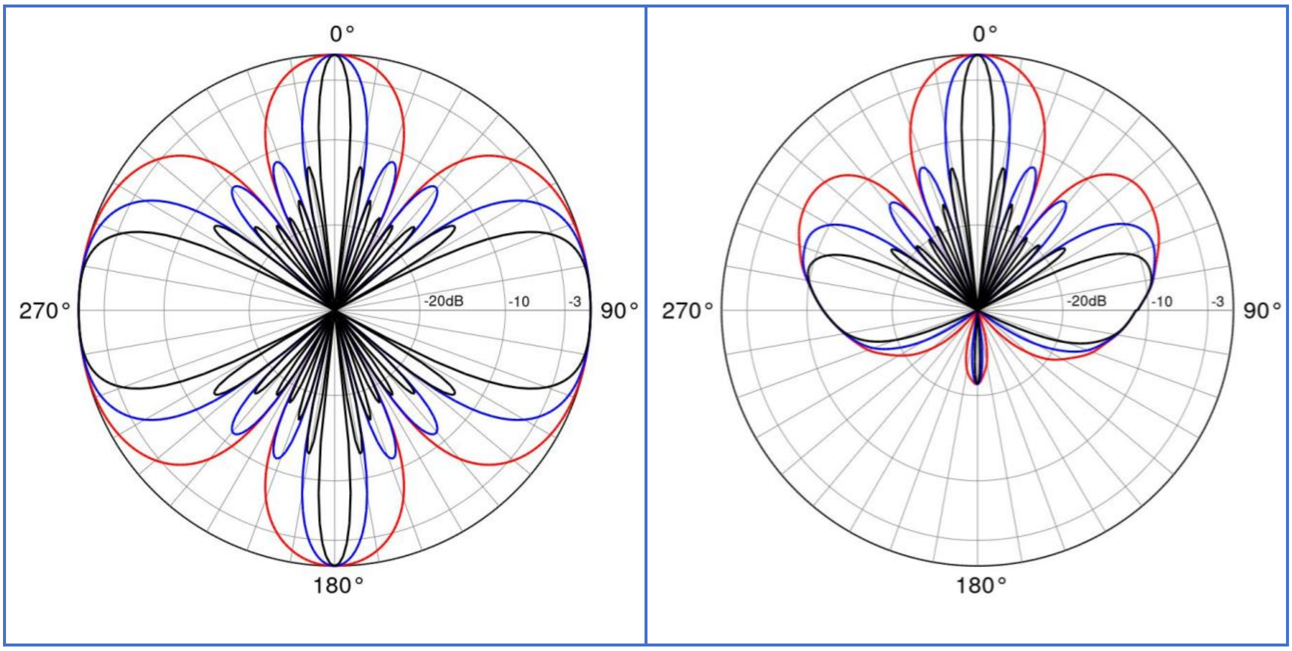


Array factor $f_g(\alpha)$ **for N=2, 4, 8**

Radiation pattern of the array $f_{array}(\alpha)$ **for N=2, 4, 8**

Figure 2.4

Array factor and radiation pattern for ***d=0.75λ***.

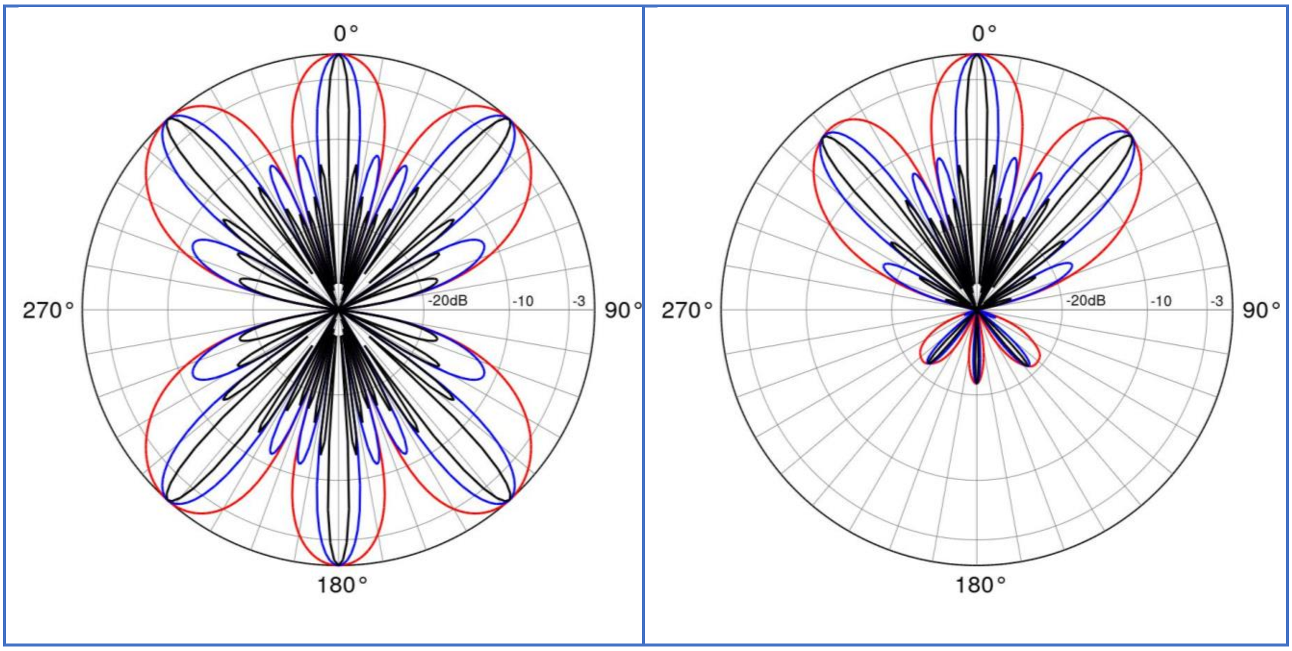


Array factor $f_g(\alpha)$ **for N=2, 4, 8**

Radiation pattern of the array $f_{array}(\alpha)$ **for N=2, 4, 8**

Figure 2.5

Array factor and radiation pattern for $d=\lambda$.

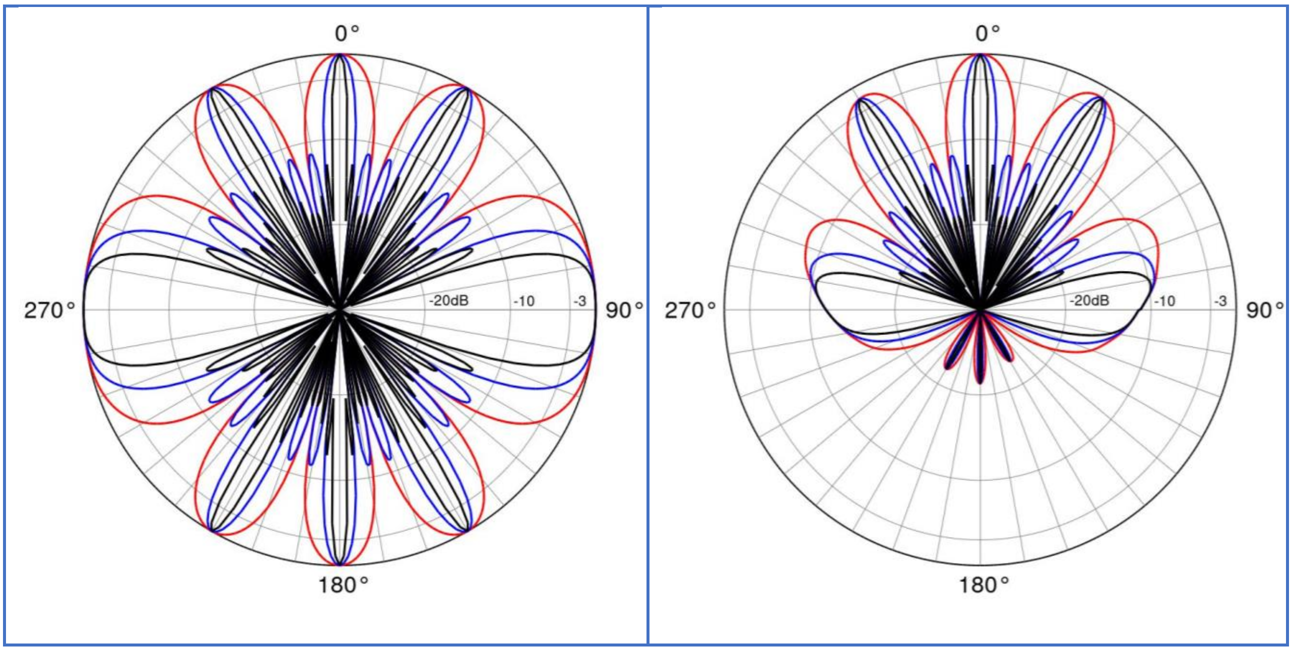


Array factor $f_g(\alpha)$ **for N=2, 4, 8**

Radiation pattern of the array $f_{array}(\alpha)$ **for N=2, 4, 8**

Figure 2.6

Array factor and radiation pattern for $d=1.5\lambda$.



Array factor $f_g(\alpha)$
for N=2, 4, 8

Radiation pattern of the array $f_{array}(\alpha)$
for N=2, 4, 8

Figure 2.7
Array factor and radiation pattern for ***d=2λ***.

When the spacing *d* is increased from *0.5λ* to *λ*, a pair of secondary lobes, equal in level to the main lobe oriented at 0°, appear in the array factor. These are known as **grating lobes** (**Figure 2.5**).

With a further increase in spacing, the grating lobes, which are symmetric respect to the main lobe, get closer and closer to it.

For *d=2λ*, a second pair of grating lobes appears, and as the distance between the individual radiating elements continues to increase, they increasingly invade the frontal region of the array (α < ±90°).

It is also interesting to note that the orientation of the grating lobes depends on *d/λ* and not on *N*.

In general, for spacing values between *mλ* and *(m+1)λ* with *m* being an integer, there are *2m* grating lobes in the range −90° ≥ α ≥ 90°, symmetrically positioned respect to the main lobe.

Figure 2.8 shows the angular positions of the grating lobes as a function of the spacing of the elementary antennas that form the array.

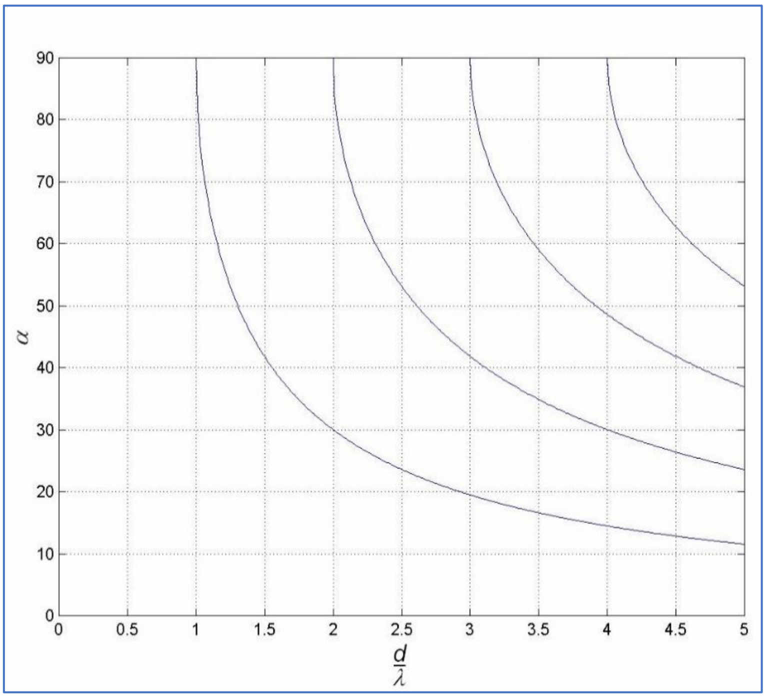


Figure 2.8
Angular position α of the grating lobes as a function of the array spacing *d/λ*.

The effect of the array factor on the radiation pattern of the array is clearly evident in the polar plots on the right side of the **Figures 2.3 ÷2.7**.

From **Figure 2.8**, it is easy to see that, for example, for $d=3\lambda$, there are three pairs of grating lobes, oriented at approximately ±20°, ±42°, and ±90° respect to the broadside direction of the array.

It follows that, to avoid strong side lobes and the consequent decay in directivity, the lobe of the elementary antenna that forms the array must be much narrower than 40°, and any of its potential secondary lobes, if not well attenuated, should not coincide with the grating lobes at ±42° and ±90°.

3. Conclusions.

What has been discussed so far sets specific constraints on the design of an array antenna, concerning the maximum achievable performance (gain and side lobes) in relation to the level of complexity (number of elements N) required to achieve it.

Let's summarize them in a few key points.

- The spacing d, used in the vast majority of arrays, is between 0.5λ and λ. Spacing values smaller than 0.5λ are impractical both in terms of the physical size of the individual radiating element and due to interaction issues between the individual sources in the array, which must maintain a certain degree of decoupling. Values greater than λ are impractical if the array elements have a rather wide beam and are usually employed when coupling individual antennas that already have fairly narrow beams, such as Yagi or dish antennas. However, in this case, it is crucial to ensure that the angular position of the secondary lobes of the individual antennas does not coincide with the orientation of the grating lobes caused by the array factor.
- For a given number N of elements in the array, increasing the spacing d allows for an increase in the total size of the array and, consequently, its directivity, narrowing the main lobe. This leads to a limiting value of spacing, dependent on $f_{el}(\theta,\phi)$, at which the maximum directivity of the array is achieved. Beyond this value, side lobes appear due to grating lobes, which reduce the energy focused in the main beam.
- As N increases, the array factor becomes an increasingly "sharper" function, and the beamwidth of the individual source is no longer significant in determining the width of the array's main lobe.
- Given a certain total size $L_\lambda=(N-1)d/\lambda$ of the array, the value of N determines the quantity and behaviour of the side lobes.

In this discussion, we have only considered uniformly illuminated arrays, where each radiating element is fed with equal power and phase.

In a future technical analysis, we will explore the effect of a relative phase shift between the individual elementary antennas in the array, as well as the effect of non-uniform illumination. This will allow us to see how it is possible to develop a custom array antenna for a specific application.

We will then discuss the beam synthesis of an array.

*All the information and experiences presented in this article are the result of the design, development, and production of custom professional antennas carried out by **ElettroMagnetic Services Srl** using the **AntennaCustomizer** method.*

*For questions, clarifications, or further information on this or other topics related to professional antennas, please contact **bollini@elettromagneticservices.com**.*

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